Branching Quantifiers, Functional WH and List Answers of WH-Questions

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Wh-questions are answered with a list of pairs when quantified expressions such as "every professor" interact with wh-phrases. The theory of branching quantification is extended to wh-quantifier interactions to account for the question reading associated with list answers. It is also shown that there are several phenomena which would be better explained when functional variables are incorporated in the representation of the list reading.

0. Introduction
This paper discusses the list reading associated with questions such as in (1). The question is answered in at least three ways: (i) by naming the student every professor saw e.g. "John" as in (1a); (ii) by expressing the type of student every professor saw, e.g. "his advisee" as in (1b); and (iii) by presenting professor-student pairs such that the former saw the latter, e.g. "Prof. Smith saw Mary, Prof. Jones, Kate, Prof. Li, Bill" as in (1c) It is assumed that these three types of answers correspond to three different readings or meanings of the question. The list reading is the one associated with the answer in (1c).

(1) Which student did every professor see?
   (a) Every professor saw) John -individual answer
   (b) (Every professori saw) his/heri advisee -functional answer
   (c) Prof. Smith saw Mary, Prof. Jones, Kate, Prof. Li, Bill,.... -list answer

Our concern here is to provide an adequate semantic representation for wh-questions corresponding to list answers. We first review the analyses in [1, 2]. Next, it will be shown that the list reading is also defined through branching quantification. Finally, we consider the possibility of implementing functional variables in the semantics of branching quantifiers.

Throughout the paper, we assume the semantics of questions proposed in [3,4], in which questions are considered as a collection of possible answers to them. Under this approach, the question in (2a) is , for example, considered as the set of propositions in (2b). Appropriate answers are to be a subset of this set. (2b) can be stated as in (2c) in Montague's Intensional Logic.

(2) a. Which student did Prof. Smith see?
   b. {that Prof. Smith saw a1, that Prof. Smith saw a2, ...that Prof. Smith saw an} where a1, a2,....., an are the students in the domain
   c. \( \lambda p \exists x [s'(x) \land p=^\wedge \text{see}'(\text{Prof. Smith}', x)] \)

[1, 2] present semantic analyses of the list reading, utilizing a 'minimal witness set' of quantifiers. A 'witness set' of a quantifier Q is defined in [5] as follows.
(3) witness set w of Q is (i) a subset of the set a Q lives on; (ii) w ∈ Q

For example, the witness set of "every professor" is the set of professors; "no professor" has an empty set as its unique witness set; any set which contains 'few' numbers of professors is a witness set of "few professors". A minimal witness set of Q is the 'smallest' witness set; the witness set which is not included by any other witness sets of Q. Thus, in case of 'few professors', for instance, the minimal witness set is the empty set.

Consider the question in (1) again. The list interpretation of (1) is roughly associated with the translation in (1'a) in the approach in [1].

(1') a. λp∃x∃y[x ∈ a minimal witness set of "every professor" ∧ st'(y) ∧
   p=∧see'(x,y)]
   b. { that a1 saw b1, that a1 saw b2,......, that a1 saw bm,
       that a2 saw b1, that a2 saw b2, ......, that a2 saw bm,
       ......
       that an saw b1, that an saw b2, ...... that an saw bm }
   where a1,...,an are the professors and b1,...,bm are the
   students in the domain

The minimal witness set that "every professor" lives on is the set of all the professors. We make a list, picking out a member of the professor-set and then giving each member a student s/he saw. We are concerned with the propositions of the form "that a saw b" as in (1b). Interestingly, the availability of the list reading, but not other readings, varies with the quantifier used in wh-questions. Consider the question in (4a). The question does not seem to have a list answer. (4b) exemplifies the type of list answer to the question in (4a) that we might expect to be possible, but in fact (4b) is not an appropriate answer, as * indicates.

(4) a. Which student did no professor(s) see?
   b. * Prof. Smith didn't see Bill, Prof. Jones didn't see Kate,

The question in (5a) contains a quantified term "few professors", and this question does not allow list answers, either.

(5) a. Which student did few professors see?
   b. Prof. Smith saw Mary, Prof. Jones, Kate, Prof. Li, Bill,

We have seen that wh-questions with "every N" have list answers while "no N" and "few N" do not have list answers. [1, 2] explain the contrast as follows. The minimal witness set that "no professor(s)"or "few professors" lives on is an empty set which does not have any member in it. Consequently, we cannot extract any member from it and we cannot make a list.

A potential problem arises when we consider how to derive (1') in a natural manner. Specifically, "existential" quantification should be somehow introduced when "every professor" is interpreted in (1'). It has been proposed that wh-terms are naturally assumed to be associated with existential quantification (See [6]). However, "every professor" is usually associated with universal quantification. It should be noted here that there is of course a
reason why universal quantification is not used at the proposition level. Consider the formula in (6a).

(6)  

a. \( \lambda p[\forall x(\text{prof}'(x)) \rightarrow \exists y(\text{st}'(y)) \land p = \text{see}'(x,y)] \)

b. \( a_1, a_2 = \text{professors}, b_1 = \text{student} \rightarrow p = \text{see}'(a_1, b_1) = \text{see}'(a_2, b_1) \)

As [6] points out, if there is more than one professor in the world, this formula will be false; if there are professor \( a_1 \), and \( a_2 \) and the student \( b_1 \) in the world, it should be the case that \( \text{see}'(a_1, b_1) = \text{see}'(a_2, b_1) \) since \( p \) is a rigid designator.

The question now is if there is an other way to represent the list interpretation of questions nicely. I will propose an alternative way of representing the list interpretation, utilizing a mechanism to define "cumulative interpretations" in the next section.

2. Branching Quantification
Let us first make clear what a "cumulative interpretation" is first. Consider the sentence in (7).

(7) Three professors saw four students.

a. wide scope; three professors
b. wide scope; four students
 narrow scope; four students
 narrow scope; three prefects

(7) has scopal readings; either "three professors" or "four students" takes wide scope relative to the other as in (7a) and (7b). It also has a "cumulative reading", which is a non-scopal reading that only involves a group of three professors and a group of four students such that; (i) any member of the professor group saw at least one member of the student group; and that (ii) any member of the student group was seen by at least one member of the professor
group. Situations associated with this reading are exemplified in (7c).

[7] proposes the translation for this reading shown in (7'), developing the idea seen in [8].

\[(7') \quad \text{three (prof')} \]
\[
\text{see } = \text{def.} \exists X \subseteq \text{(prof')} \exists Y \subseteq \text{(st')} \{ \text{three (prof')} X \land \text{four (st')} Y \}
\]

\[(7') \quad \text{four (st')} \]
\[
\text{& } \exists x \in X \rightarrow \exists y \in Y \{ y \in Y \land \text{see}(x,y) \}\} \land \forall y \in Y \rightarrow \exists x \in x \subseteq X \land \text{see}(x,y)\}
\]

(7') roughly says that there is a witness set of three professors (i.e. the set of three professors) X and a witness set of four students (i.e. the set of four students) Y that satisfies the following condition; each member of X stands in a relation to some element of Y, and for each element of Y there is some element of X (henceforth, we will call this condition the 'each-some/some-each condition', following [7]; we will also use it in the definition for expository purposes). The essential part of this analysis is 'branching quantification'; two quantifiers are associated with their witness sets; the relation is specified between members of the one set and members of the other set.

Now we can detect some similarity between the "list interpretation" of wh-questions such as in (1) and the "cumulative interpretation" of sentences such as in (7). We have seen how the list interpretation may be analyzed adopting the notion 'minimal witness set'. The cumulative reading may be analyzed using branching quantification which involves 'witness sets' of two or more quantifiers. The reason why 'minimal' witness sets instead of witness sets is used in [1,2] is that monotone decreasing quantifiers such as "few professors" can have witness sets with some members while their minimal witness sets are empty sets. We have seen that wh-questions with "few professors" such as in (5) are not answered with a list. If a 'witness set' is a relevant set in the semantic context, we can make a list out of the members of the set and we expect the list answer to be available.

It should be noted here that [7] shows that branching quantification for monotone-decreasing quantifiers such as "no N" or "few N" are not straightforwardly defined in the semantics (see [5] for monotonicity). [8] ends up proposing two different definitions of branching quantifiers; one for monotone-increasing quantifiers (such as "every N" or "some N") and another for monotone-decreasing quantifiers. There is no way to define branching quantifiers when quantifiers involved are different in monotonicity. [8] points out that interpretations associated with branching quantifiers are difficult to obtain when sentences contain both monotone-increasing and monotone-decreasing quantifiers as in (8).

(8) ? Few of the boys in my class and most of the girls in your class have dated each other.

Consider wh-questions with quantifiers. Existential quantifiers, which wh-terms are often associated with, are monotone-increasing quantifiers. Wh-questions with "few professors" contain both monotone-increasing and decreasing quantifiers. Thus, if the branching-type of quantification is involved at all in the list reading of the question, there seems to be a way to explain the lack of the list reading in connection with difference in
Branching Quantifiers and WH-Questions

monotonicity. We do not have to refer to "minimal" witness sets to account for the availability of the list reading.

The proposed analysis for the list reading in (1) is as follows.

(1) Which student did every professor see?

(9) a. $Q$(every $(\text{prof})$)((some$(\text{st}')$(see$(x,y)$))
    =def. $\lambda p\exists X \subseteq \text{(prof)} \exists Y \subseteq \text{(st')}[\text{every}(\text{prof})X \& \text{some}(\text{st'})Y$
    & $p=X$ and $Y$ satisfy each-some/some-each condition]

b. $(\text{that} \{a_1, a_2 ... a_n\} \text{ see} \{b_1\}$, $(\text{that} \{a_1, a_2 ... a_n\} \text{ see} \{b_1, b_2\}$,...
    that $(\text{that} \{a_1, a_2 ... a_n\} \text{ see} \{b_1, b_2, ...........b_m\})$
    $a_1,...,a_n$ are the professors and $b_1,...,b_m$ are the students

c. $(\text{that} \{a_1, a_2, a_3\} \text{ see} \{b_1, b_2\})$ $a_1, a_2, a_3$ are the professors

d. $a_1 \xrightarrow{\text{see}} b_1$
    $a_2 \xrightarrow{\text{see}} b_2$

  e. answerhood condition; $A(p, Q)$ iff "$p$ & $\exists q \in Q [\forall p \text{ entails } q]"

We assume the operation $Q$ which takes a sentence and one quantifier and one wh-phrase, which is treated as an existential quantifier. Through this operation, we get a set of propositions of the form that there is an "each-some/some-each" seeing-relation (henceforth, see) between the witness sets of "every professor" and "some student" (that is, a set of all the professors and a student-set with at least one member in it). For example, in the world where there are the three professors $a_1$, $a_2$ and $a_3$ in the world and $b_1$ and $b_3$ are students, the proposition in (9d) "that $\{a_1, a_2, a_3\} \text{ see} \{b_1, b_2\}" is a possible proposition. This proposition cannot be an appropriate answer as it is. As [9] argues in a similar context, the principle of cooperation requires a more informative answer. When the relevant situation is "$a_1$ saw $b_1$ and $a_2$ saw $b_2$ and $a_3$ saw $b_2$" as in (9d), we are obliged to spell out the situation, saying "$a_1$ saw $b_1$ and $a_2$ saw $b_2$ and $a_3$ saw $b_2$" in order to be sufficiently informative for the current purpose of exchange. Alternatively, we can take the strong position that semantic answerhood requires a listing of the information pertinent to questions with the list interpretation as in (9e). (9e) says that a proposition $p$ is a (complete) answer of a question $Q$ iff $p$ is true in a particular world and it entails a member of question meaning $Q$.

3. Functional variables and wh-questions

The present analysis has some consequences for syntactic analyses of wh-quantifier interactions. It is observed that the list reading is available only when some syntactic requirement(s) is/are met. For example, [10] observes that the questions as in (10) cannot be answered with a list while the questions as in (1) have a list answer.

(1) Which student did every professor see?
(10) Which professor saw every student?

  -* Prof. Smith saw Bill, Prof. Jones saw Kate,...
[10] explains this asymmetry roughly as follows; (i) the list reading is obtained when a quantifier has wide scope relative to a wh-element; (ii) scope order is decided based on hierarchical information at LF; (iii) due to a syntactic constraint (namely, the PCC), a quantifier may not be in the position that leads to the wide scope interpretation. The analysis proposed here does not touch on the scope relation between a quantifier and a wh-term in the usual manner. It rather states that a quantifier and a wh-term are not scopally related at all. They or their witness sets are treated equally. Consequently, we cannot adopt the analysis in [10] for the subject-object asymmetry seen in (1) and (10). Remember, however, that quantifying in at the proposition level is problematic in semantics. It is not desirable from the semantic point of view to explain the availability of the list reading by associating 'being in a higher hierarchical position than a wh-term' with 'having wide scope interpretation' or 'quantifying in after the formation of question-propositions'.

The asymmetry is explained if we assume that wh-terms are associated with a 'function' in the list interpretation, following [2,11]. Consider (1c), which is an example of a so-called functional answer to the question in (1).

(1c) (Every professor saw) his/her advisee.

According to [1,11], the reading of the question (1) corresponding to the answer in (1c) roughly asks "which function gives, for a professor in the domain, a student in the "seeing" relation with the professor". The answer in (1c) says that "advisee" function, which relates an adviser to his/her advisee, is such a function. [2] argues that the derivation as in (11) can be involved in the functional interpretation of the questions such as in (1).

(11) (for (1))
   a. Which \( f \) [every professor saw \( f(x) \)]
   b. for which \( f \): every professor [x saw \( f(x) \)]

Note that in (11b) there is a functional variable \( f \) applied to the individual variable \( x \) in the wh-trace position; the individual variable \( x \) is bound by the quantifier in the subject position; the functional variable \( f \) is bound by the wh-operator.

Consider the question in (10) again. (10) does not have functional answers such as in (12).

(10) Which professor saw every student?
(12) His/Her adviser. (cf. *His/Her adviser saw every students)

[2] shows that the lack of the functional answers in (14) can be explained as an instance of a Weak Crossover(WCO) violation. Consider the derivation in (13).

(13) (for (10))  a. Which \( f \) [\( f(x) \) saw every student]  
   b. for which \( f \): every student [\( f(x) \) saw x]

In (13b), "every student" must cross \( x \) of \( f(x) \) to bind that \( x \).
Branching Quantifiers and WH-Questions

[2,11] claim that list readings are a special case of functional readings; functions are sometimes described with a list of pairs as in (14).

\[
(14) \quad f = \begin{cases} 
\text{Prof. Smith} \rightarrow \text{Mary} \\
\text{Prof. Jones} \rightarrow \text{Kate} \\
\text{Prof. White} \rightarrow \text{John}
\end{cases}
\]

In [2], a function of this type is called an 'extensional' function, in contrast with the function associated with a functional answer, which is referred as 'intensional' function. Some examples of intensional/extensional functions are given below, using addition of numbers.

\[
(15) \quad \begin{array}{ll}
\text{a. intensional} & \text{b. extensional} \\
& \begin{cases} 
f(x)=x+1 \\
f = 1 \rightarrow 2 \\
2 \rightarrow 3 \\
3 \rightarrow 4 \\
\end{cases}
\end{array}
\]

[2] shows that the unavailability of the list reading in (11) is also explained in terms of the WCO violation, assuming that \textit{wh}-terms are associated with functional variables even in the case of the list reading.

Functional variables can be technically implemented in the semantics of the list reading proposed here. Remember that in the operation for the list reading shown in (9), we have focused on relations between a witness set of "every professor" and a witness set of "some student". We are now interested in the relation between a witness set of "every professor" and a witness set of "some function which gives out a student". The relevant operation for the list reading of (1) is now illustrated as in (16).

\[
(1) \quad \text{Which student did every professor see?}
\]

\[
(16) \quad Q(\text{every}(\text{prof}))(\text{(some}(f(\forall x f(x)(st')))(\text{see}(x,f(x))))
\]

\[
= \text{def. } \lambda p \exists x [x \subseteq (\text{prof}) \land \exists y \subseteq (\{f: \forall x f(x)(st')\})]\{\text{every}(\text{prof}) \land 
\]

\[
\land \text{some}(\{f: \forall x f(x)(st')\}) \land p = x \land F \text{ satisfy each-some/some-each condition}
\]

(16) contains all the propositions of the form: the set of all the professors (i.e., the witness set of "every professor") and a witness set of "some student-function" holding a relation such that (i) each professor saw a student supplied through \textit{some} function in the witness set of "some student-function" and (ii) each function of the witness set of "some student-function" takes \textit{some} professor and gives out a student whom the professor saw. The relevant relation will be abbreviated as \textit{see} \textit{f} hereafter.

Assume that there are professors a1, a2, a3 and student-functions f1, f2, f3. We have a set of propositions associated with the question as in (17a).

\[
(17) \quad \text{professors a1, a2, a3; student-functions f1, f2, f3}
\]

\[
\text{a. } \{ \text{that } \{a_1,a_2,a_3\} \text{ see } f_1 \}, \{ \text{that } \{a_1,a_2,a_3\} \text{ see } f_1 f_2 \}, \{ \text{that } \{a_1,a_2,a_3\} \text{ see } f_1 f_3 \}, \{ \text{that } \{a_1,a_2,a_3\} \text{ see } f_2 f_3 \}, \{ \text{that } \{a_1,a_2,a_3\} \text{ see } f_1 f_2 f_3 \}
\]
In (18a) below, one of the propositions that (17a) contains is picked out to illustrate the point clearly. (18a) will correspond to the set of propositions in (18b). If $f$ is a function which maps professors to students as in (18c), (18b) corresponds to a set of propositions in (18d).

\[(18)\]

\[
\begin{align*}
\text{a.} & \quad \{a_1, a_2, a_3\} \text{ see } f \{f_1\} \\
\text{b.} & \quad \{\text{that } a_1 \text{ saw } f_1(a_1), \text{ that } a_2 \text{ saw } f_1(a_2), \text{ that } a_3 \text{ saw } f_1(a_3)\}
\end{align*}
\]

\[
\begin{align*}
\text{c.} & \quad f_1 = a_1 \rightarrow b_1 \\
& \quad a_2 \rightarrow b_2 \\
& \quad a_3 \rightarrow b_3 \\
\text{d.} & \quad \{\text{that } a_1 \text{ saw } b_1, \text{ that } a_2 \text{ saw } b_2, \text{ that } a_3 \text{ saw } b_3\}
\end{align*}
\]

It is possible that each professor is related to different student-functions as in (19a). (19b) is an example of what (19a) could correspond to.

\[(19)\]

\[
\begin{align*}
\text{a.} & \quad \{a_1, a_2, a_3\} \text{ see } f \{f_1, f_2, f_3\} \\
\text{b.} & \quad \{\text{that } a_1 \text{ saw } f_1(a_1), \text{ that } a_2 \text{ saw } f_2(a_2), \text{ that } a_3 \text{ saw } f_3(a_3)\}
\end{align*}
\]

\[
\begin{align*}
\text{c.} & \quad f_1 = a_1 \rightarrow b_1 \\
& \quad a_2 \rightarrow b_2 \\
& \quad a_3 \rightarrow b_3 \\
\text{d.} & \quad \{\text{that } a_1 \text{ saw } b_1, \text{ that } a_2 \text{ saw } b_3, \text{ that } a_3 \text{ saw } b_2\}
\end{align*}
\]

We have seen that we can possibly implement functional variables in the definition of the list reading. However, we have also seen that list reading can be defined without functional variables. It will be shown below that the definition with functional variables is attractive on several counts.

First, the contrast in (1) and (10) can be explained as a consequence of WCO just as in [2]. Remember that we don't have to refer to "scope" to explain the contrast in the WCO approach, which is desirable under the proposed definition in which two quantifiers are scopally independent.

Second, consider (20), which is sort of a mixed case of the list answer and the functional answer.

\[(20)\]

\[
\text{Prof. Smith saw his advisee, John, Prof. Jones saw her TA, Bill, Prof. Lee saw his RA, Kate.}
\]

This natural language example can be associated with the proposition in (18b) above. Consider the possibility that we know not only an "extensional" aspect of functions as in (18c), but also an "intensional" aspect of them as in (21).

\[(21)\]

\[
f_1(x) = x's \text{ advisee}; f_2(x) = x's \text{ TA}; f_3(x) = x's \text{ RA}
\]

We can spell out the relation between professors and students they saw, using both intentional and extensional functions as in (20).

A final discussion concerns sentences in which a wh-term and a quantifier have an asymmetrical binding relation. Consider (22).
(22) Which student who took his/her class did every professor see?
- Prof. Smith saw John (who took his class), Prof. Jones saw Kate (who took her class), ....

In (22), the binding of "his/her" by "every professor" is possible in the list interpretation. We have defined the list interpretation through branching quantifiers. Remember that quantifiers in branching quantification are scopally independent. Nevertheless, the two quantifiers involved in (22), namely "every professor" and "some student who took his/her class" have a dependency with respect to binding. We will see that the dependency between scopally independent elements can be expressed through introduction of functional variables. Developing the ideas seen in [1,11], the question in (22) is expressed as in (22').

(22') \( Q(\text{every (prof}}))(\text{(some(f(\forall x(f(x)(\text{student-taking-class-of-x'}))))(see(x,f(x))))} = \text{def. } \lambda p \exists x \subseteq (\text{prof}) \exists f \subseteq ((f; \forall x(f(x)(\text{student-taking-class-of-x'})))) \[\text{every(prof')}X \& \text{some} ((f; \forall x(f(x)(\text{student-taking-class-of-x'}))))F & p= ^\wedge [\forall x \in X \rightarrow \exists f [f \in F \& \text{see}'(x,f(x))]] & [\forall f \in F \rightarrow \exists x [x \in X \& \text{see}'(x,f(x))]]\]

When taking a witness set, we are concerned with a function \( f \) which gives out, for any \( x \), some student who took class of \( x \). The domain of \( f \) is only indirectly constrained; \( x \) of \( f(x) \) in "see' (x, f(x))" is restricted to an element of the witness set of "every professor".

There are, however, some examples which are worrisome with respect to the binding involved in scopally independent elements and its relevance to the functional variables. The discussion will be open-ended, but let me at least point out the problem.

Consider (23).

(23) Which student did every professor who read his paper see?

Interestingly, (23) do not have a list answer, contrasted with (19). In (19), a \( w_h \)-phrase contains a variable to be bound and binding is possible due to a functional variable associated with a \( w_h \)-phrase. In (23), a quantifier, but not a \( w_h \)-phrase, contains a pronoun to be bound. Quantifiers are not associated with functional variables so far. Thus at this point we could say that in (23) there is no way to construe the binding relation when there is no scopal interaction between a \( w_h \)-phrase and a quantifier with a pronoun to be bound.

Now consider (24).

(24) Three professors saw fifteen students who took their class(es).

The fact is that we can observe the asymmetrical binding relation between quantifiers involved in the cumulative reading. (24) is possibly associated with the situation in (24'). In (24a), binding of "their" by "three professors" is admitted in the cumulative interpretation (see[12] for the possible application of functional variables to quantifiers).
We have fallen into a dilemma; if we allow functional variables for quantifiers as well as wh-phrases, we could explain the binding relation involved in the cumulative reading in the same way we do for the binding relation involved in the list reading. However, if we do so, the contrast in (22) and (23) has to be explained in some other way.

References